CERTAIN FAMILIES OF DISTRIBUTION SERIES RELATED TO Q-CALCULUS

The Synopsis for the registration of Ph.D. in

MATHEMATICS

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2021

1 Introduction

Theory of special functions has a long and varied history with immense literature due to their applications in solving various problems arising in physical, biological and engineering sciences. Special functions have an origin in the solution of partial differential equations satisfying certain prescribed conditions. At present special functions are defined in several ways notably by power series, generating functions, infinite products, integrals, difference equations, trigonometric or orthogonal function series. Eminent mathematicians notably Euler, Legendre, Gauss, Jacobi, Weierstrass, Kummer, Riemann, Ramanujan worked hard to develop special functions like Bessel functions, Whittaker functions, Gauss hypergeometric function and the polynomials that go by the names of Jacobi, Legendre, Laguerre, Hermite etc. The Gaussian hyper geometric function pFq and its special cases are commonly used in applied mathematics and mathematical physics.

Fractional Calculus is a field of mathematic study that grow out of the traditional definitions of the calculus integral and derivatives operators in much the same way fractional exponents is an outgrowth of exponents with integer value. Consider the physical meaning of the exponent. According to our primary school teachers exponents provide a short notation for what is essentially a repeated multiplication of a numerical value. This concept in itself is easy to grasp and straight forward. However, this physical definition can clearly become confused when exponents of non integer value.

The fractional derivatives (and fractional integral) of special function of one and more variable is important such as in the evolution of series and integrals the derivation of generating function and the solution of the integral equation motivated by these and many other avenues of applications the fractional calculus operator is much used in the theory of special function of

one and more variables.The fractional derivative, extension of the ordinary derivative to an non integral value of the order, is of immense utility in finding the solution of ordinary, partial and integral equations, as well as in other contexts. Fractional derivatives for conventional functions were introduced long before the systematic development of the generalized function. Indeed one of the more creditable aspects of the new generalized functional theory would seem to be the fractional derivatives as well as hypergeometric function.

The formulation of fractional calculus began shortly after the classical calculus was established. Since its definition is based on the concept of a noninteger order either integral or derivative, the fractional calculus had been considered as a subject in pure mathematics with no real applications for a long time. However, the role of fractional calculus has been changed in recent decades. Its applications take place in many fields ofmathematical sciences. Extended from the fractional calculus, the fractional q-calculus is the q-extension of the ordinary fractional calculus. Many results of the study on theory of q-calculus operators in recent decades have been applied in various areas such as problems in the ordinary fractional calculus, optimal control, solutions of q-difference equations, q-differential equations, q-integral equations, and q-transform analysis and also in the geometric function theory of complex analysis.

In the field of geometric function theory, various subclasses of analytic functions have been studied from different viewpoints. The fractional q-calculus is the important tools that are used to investigate subclasses of analytic functions. For example, the extension of the theory of univalent functions can be described by using the theory of q-calculus.In [15], Ismail et al. introduced the generalized class of starlike functions by using the q-

difference operator and replaced the right-half plane by a suitable domain. In a similar way, Agrawal and Sahoo [1] introduced the generalized class of starlike functions of order q and Purohit and Raina [21], Raghavendar and Swaminathan [22] also introduced the class of q-analogue to close-to-convex functions.Moreover, the q-calculus operators, such as fractional q-integral and fractional q-derivative operators, are used to construct several subclasses of analytic functions.

In addition, the differential operators have been extensively investigated in the field of geometric function theory. The well-known differential operator defined on the class of analytic functions is introduced by Salagean [27]. This operator was successfully used by many authors and it led to the investigation of several properties of certain known and new classes of analytic functions . However, there are many generalized Salagean operators defined by several authors. In [2], Al-Oboudi defined the generalized Salagean operator by using the technique of convolution structure used the extension of fractional derivative and fractional integral to define linear multiplier fractional differential operator.

Selvakumaran et al. [28] introduced the fractional q-differintegral operator by using the fractional q-calculus operators involving the generalized Al-Oboudi operator [2].

2 Objectives

We propose to study in the following areas:

(i). To solve fractional Mittag Lefflor equations using integral transforms.

(i). The study of q- analogues of various integral transforms such as q-Laplace transform, q-Mellin transform, q- Fourier transform, q- Henkel

transform etc. and find out their possible new applications.

(ii). To study of q-analogues on various generalization of fractional differential and integral operators.

(iii). To study of q-analogues on various generalization of coefficient bounds fractional differential and integral operators.

(iv). To obtain Hadamard product results, partial sums and neighborhood properties of subclasses.

(v). To find coefficients bounds of bi-univalent and multivalent functions involving fractional calculus operators.

(vi). To obtain coefficient estimates,distortion bounds for the functions of these classes.

(vii). To obtain certain inclusion relations and their applications by using subordination.

(viii). To find various applications of special polynomials like Geganbauer, Lucas, Pascal ,Borel, Binomial distribution series.

(ix). To find bounds of generalised multiplier transformations.

(x). To find neighborhoods of a certain class of analytic functions with negative coefficients.

(xi). To study of Sandwich-type theorems for a class of multiplier transformations associated with the Noor integral operators.

(xii). To study Geometric properties of generalized polylogarithm functions.

(i). To examine the applications of these subclasses.

3 Applications of Fractional Calculus

The subject of fractional calculus has applications in diverse and widespread fields of engineering and science such as electromagnetics, viscoelasticity, fluid mechanics, electrochemistry, biological population models, optics, and signals processing. It has been used to model physical and engineering processes that are found to be best described by fractional differential equations. The fractional derivative models are used for accurate modelling of those systems that require accurate modelling of damping. In these fields, various analytical and numerical methods including their applications to new problems have been proposed in recent years. This special issue on Fractional Calculus and its Applications in Applied Mathematics and Other Sciences is devoted to study the recent works in the above fields of fractional calculus done by the leading researchers.

Mathematical modelling of real-life problems usually results in fractional differential equations and various other problems involving special functions of mathematical physics as well as their extensions and generalizations in one or more variables. In addition, most physical phenomena of fluid dynamics, quantum mechanics, electricity, ecological systems, and many other models are controlled within their domain of validity by fractional order PDEs. Therefore, it becomes increasingly important to be familiar with all traditional and recently developed methods for solving fractional order PDEs and the implementations of these methods.

4 Importance of Study

It is expected that most of the results which will obtain in this survey may find applications in the solution of certain fractional order differen-

tial and integral equations arising problems of physical science, engineering and statistics.The subject of fractional calculus has applications in diverse and widespread fields of engineering and science such as electromagnetics, viscoelasticity, fluid mechanics, electrochemistry, biological population models, optics, and signals processing.The fractional calculus (FC) generalizes the operations of differentiation and integration to non-integer orders. FC emerged as an important tool for the study of dynamical systems, since fractional order operators are non-local and capture the history of dynamics. Moreover, FC and fractional processes have become one of the most useful approaches to deal with particular properties of (long) memory effects in a myriad of applied sciences. Linear, nonlinear, and complex dynamical systems have attracted researchers from many areas of science and technology, involved in systems modelling and control, with applications to real-world problems. Despite the extraordinary advances in FC, addressing both systems modelling and control, new theoretical developments and applications are still needed in order to describe or control accurately many systems and signals characterized by chaos, bifurcations, criticality, symmetry, memory, scale invariance, fractality, fractionality, and other rich features. The Special Issue focuses on original and new research results on fractional calculus in science and engineering. Manuscripts addressing novel theoretical issues, as well as those on more specific applications, are welcome. Topics of interest include (but are not limited to): fractional calculus theory, methods for fractional differential and integral equations, nonlinear dynamical systems, advanced control systems, fractals and chaos, complex dynamics, evolutionary computing, finance and economy dynamics, biological systems and bioinformatics, nonlinear waves and acoustics, image and signal processing, transportation systems, geosciences, astronomy and cosmology, nuclear

physics, fractional modeling in econophysics, and fractional modeling for time series.

5 Review of Literature

A Fractional calculus has its origin in the question of the extension of meaning. A well known example is the extension of meaning of real numbers to complex numbers, and another is the extension of meaning of factorials of integers to factorials of complex numbers.

Koebe [19] pioneered the study of univalent functions in 1907, when he discovered that the range of all functions in S contains a common disc $|w| < b$, where b is an absolute constant. The Koebe function demonstrates that $b \leq 1/4$. Bieberbach [5] established that b can be taken to be 1/4, and he also demonstrated that if S, then equality occurs only for rotations of the Koebe function. Because the Koebe function appears to be an external function for so many problems in class S, Bieberbach [5] suspected that it maximises for every n, resulting in the famous Bieberbach conjecture, first proposed in 1916.

For many years, this famous problem served as a challenge, inspiring the development of important concepts and ingenious techniques that now serve as the foundation of the entire subject. Bernardi [3], Hayman [14], and Goodman [10] have reviewed various developments in the theory of univalent functions. Certain aspects of the subject have also been addressed in books by Nehari [20], Goluzin [9], and Jenkins [16]. Bernardi's [4] exhaustive Bibliography of schlicht functions contains many references available in the literature on the theory of univalent functions up to 1981. Rusheweyh's book [26] provides an overview of all the results in convolution theory for univalent functions. Robertson [23] introduces the concept of order of Starlikeness as

well as the concept of convex functions of order. Still another subclass of Starlike functions that has occupied wide attention of workers in the field in recent years is the class of strongly Starlike functions introduced by Brannan and Kirwan [6] and Stankiewicz [30].

6 Basic concepts

Let A denote the class of all functions $\theta(z)$ of the form

$$\theta(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

in the open unit disc $E = \{z \in \mathbb{C} : |z| < 1\}$. Let S be the subclass of A consisting of univalent functions and satisfy the following usual normalization condition $\theta(0) = \theta'(0) - 1 = 0$. We denote by S the subclass of A consisting of functions $\theta(z)$ which are all univalent in E. A function $\theta \in A$ is a starlike function of the order α, $0 \leq \alpha < 1$, if it satisfy

$$Re \left\{ \frac{z\theta'(z)}{\theta(z)} \right\} > \alpha, \ (z \in E).$$

We denote this class with $S^*(\alpha)$.

A function $\theta \in A$ is a convex function of the order α, $0 \leq \alpha < 1$, if it satisfy

$$Re \left\{ 1 + \frac{z\theta''(z)}{\theta'(z)} \right\} > \alpha, \ (z \in E).$$

We denote this class with $K(\alpha)$.

Let T denote the class of functions analytic in E that are of the form

$$\theta(z) = z - \sum_{n=2}^{\infty} a_n z^n, \quad a_n \geq 0 \ (z \in E)$$

and let $T^*(\alpha) = T \cap S^*(\alpha)$, $C(\alpha) = T \cap K(\alpha)$. The class $T^*(\alpha)$ and allied classes possess some interesting properties and have been extensively studied by Silverman [29] and others.

A function $\theta \in A$ is said to be in the class of uniformly convex functions of order γ and type β, denoted by $UCV(\beta, \gamma)$, if

$$Re\left\{1 + \frac{z\theta''(z)}{\theta'(z)} - \gamma\right\} > \beta \left|\frac{z\theta''(z)}{\theta'(z)}\right|,$$

where $\beta \geq 0, \gamma \in [-1, 1)$ and $\beta + \gamma \geq 0$ and it is said to be in the class corresponding class denoted by $SP(\beta, \gamma)$, if

$$Re\left\{\frac{z\theta'(z)}{\theta(z)} - \gamma\right\} > \beta \left|\frac{z\theta'(z)}{\theta(z)} - 1\right|,$$

where $\beta \geq 0, \gamma \in [-1, 1)$ and $\beta + \gamma \geq 0$. Indeed it follows that

$$\theta \in UCV(\gamma, \beta) \Leftrightarrow z\theta' \in SP(\gamma, \beta).$$

For $\beta = 0$, we get respectively, the classes $K(\gamma)$ and $S^*(\gamma)$. The function of the class $UCV(1, 0) \equiv UCV$ are called uniformly convex functions, were introduced and studied by Goodman with geometric interpretation in [11.12[and uniformly starlike functions in [12]. The class $SP(1, 0) \equiv SP$ is defined by Ronning [25]. The classes $UCV(1, \gamma) \equiv UCV(\gamma)$ and $SP(1, \gamma) \equiv SP(\gamma)$ are investigated by Ronning in [24]. For $\gamma = 0$, the classes $UCV(\beta, 0) \equiv \beta - UCV$ and $SP(\beta, 0) \equiv \beta - SP$ are defined respectively, by Kanas et al. in [17.18].

7 Methodology

In the present study, it is proposed:

1. To investigate the fractional calculus operators of the generalized Mittag-Leffler type poission distributions series Geganbauer polynomials

2. To derive the recurrence relations and integral representations of the generalized Mittag-Leffler type functions.

3. To generalize the functions of fractional calculus and derive the relations that exists among these functions and fractional calculus operators.

4. To use the integral transforms for finding the solutions of the fractional differential and integral equations.

5. To find the solution of fractional differential and integral equations arising in physical, biological and engineering sciences.

6. To deduce the results for the basic hypergeometric functions as an application of fractional q-Calculus.

8 Year-wise Scheme of the Project

From the objectives of the research project, it is clear that the said study is very vast and therefore, it is difficult to chalk out the concrete time plan for its completion. However the rough sketch of the two-year year-wise work-plan of the study of the present research project is as follows.

In the first year, the basic literature concerning the fundamental background concepts of the research project will be collected and studied with their relevance on the chosen topic. This process also includes attendance of seminars and guest lectures on the mathematical analysis and applications. In the second year of proposed plan, we shall collect the literature concerning the fractional calculus and special functions along with some literature covering the applications in Geometric function Theory. This process includes the collection of research papers along with current work of reputed mathematicians for the detailed study from various angles of conclusions.

The discussion and presentation of the new ideas will be made with other researches working in the held of undertaken research project along with the publication of the new findings in the reputed referred journals. This step also includes the document preparation of the research project for its submission to university for the award of Ph. D. degree in the subject of mathematics. The review of the research work done will be taken after every six month and will be presented before the concerned committee in the form of six monthly progress reports during the two-year tenure of the work.

9 Conclusion

The outcomes of this thesis are expected to constitute a fresh basic and unique addition to the topic of Geometric function Theory and applications. The current study is believed to be valuable to scientists, engineers, and mathematicians as well. who are delving into the universal classics in addition to anomalous occurrences regulated by univalent functions To anticipate and design, differential and integral equations are used.The study's future plan in a better way.

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